Solution to 2024-JEE Advanced Full Test-1 |Paper-1

PHYSICS

1.(3) Using conservation of energy $\frac{1}{2}mv_0^2 = \frac{-GMm}{R} + \frac{1}{2}mv^2$

$$\frac{GMm}{8R} + \frac{GMm}{R} = \frac{1}{2}mv^2 \qquad \Rightarrow \qquad \frac{9GMm}{8R} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{9GM}{4R}} = 3\sqrt{\frac{GM}{4R}}$$

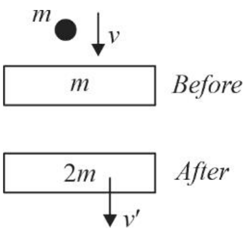
Using conservation of angular momentum

$$mv_0 d = mvR$$

$$\sqrt{\frac{GM}{4R}}d = 3\sqrt{\frac{GM}{4R}}R \quad \therefore \quad \frac{d}{R} = 3.$$

2.(3) $v = \sqrt{2gh}$

After collision



$$v' = \frac{v}{2}; \qquad W_s + W_g = \Delta k$$

$$-\frac{1}{2}k\left(\left(\frac{mg}{k}+b\right)^2 - \left(\frac{mg}{k}\right)^2\right) + 2mgb = 0 - \frac{1}{2}2mv'^2$$

$$-\frac{1}{2}k(9) + 3mg = -\frac{mv^2}{4}$$

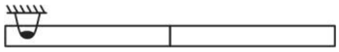
$$15 = \frac{v^2}{4} = \frac{2gh}{4} \qquad \Rightarrow \qquad h = 3\text{ m}$$

3.(1.80) As impulse exerted by wall is equal to change in momentum of the system.

$$F_{av.}\Delta t = 3m.V_{cm} = 3m.\frac{x}{3}\sqrt{\frac{k}{m}}$$

$$\Rightarrow \qquad F_{av} = \frac{x}{\Delta t}\sqrt{km} = \frac{2\pi}{\pi}\sqrt{\frac{k}{m}} \times \sqrt{km} \times \frac{1}{100} = 1.8$$

4.(0.27-0.28)



$$2Mg\frac{L}{2}+Mg\left(\frac{3L}{2}\right)=\frac{1}{2}\left((2M)\frac{L^2}{3}+\frac{ML^2}{12}+M\left(\frac{3L}{2}\right)^2\right)\omega^2$$

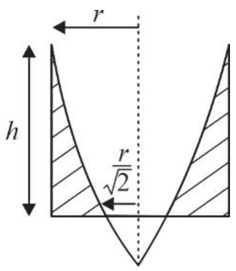
$$\frac{5MgL}{2}=\frac{1}{2}\left(\frac{2}{3}+\frac{1}{12}+\frac{9}{4}\right)ML^2\omega^2$$

$$\frac{5gL}{2}=\frac{1}{2}\left(\frac{24+1+27}{12}\right)L^2\omega^2;\quad \omega^2=\frac{5g}{3L}$$

After lower half breaks off

$$\frac{1}{2}\left(\frac{2ML^2}{3}\right)\omega^2=(2M)gh;\qquad h=\frac{\omega^2L^2}{6g}=\frac{5L}{18}=0.28\,L$$

5.(100) $h=\dfrac{\omega^2\left(r^2-\left(\frac{r}{\sqrt{2}}\right)^2\right)}{2g}$



$$\omega^2=\frac{4gh}{r^2}=10000$$

$$\omega=100\text{ rad/s}$$

6.(4) Let atmospheric pressure be P_0.

Suppose the pressure inside the tube is $6P_0$ (the maximum allowed), and the liquid comes out of the hole at speed *v*.

Since the hole is very small, the velocity of the liquid inside the tube will be negligible

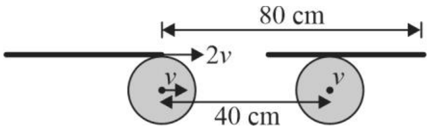
Applying Bernoulli's theorem,

$$6P_0=P_0+\frac{1}{2}\rho v^2\qquad\Rightarrow\qquad v=\sqrt{\frac{10P_0}{\rho}}=\sqrt{\frac{10\times10^5}{10^4}}=10\text{ m/s}$$

Therefore, $Q_{max}=v\times$ (area of cross-section of the hole)

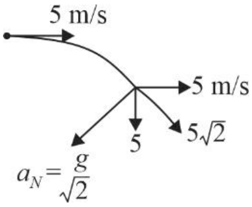
$$\Rightarrow\quad Q_{max}=(10)(4\times10^{-6})=4\times10^{-5}\ \text{m}^3/\text{s}$$

7.(40) $\int v dt = 40 \text{ cm} \Rightarrow \int 2v\, dt = 80\text{cm}$



$\therefore \quad \ell = 80\text{cm} - 40\text{cm} = 40\text{cm}$

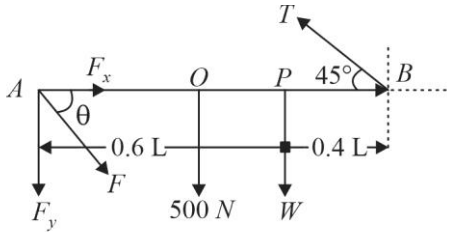
8.(7.07)



$R = \frac{v^2}{a_N} = \frac{(5\sqrt{2})^2}{5\sqrt{2}} = 5\sqrt{2} = 7.07$

9.(BD) *AB* – beam of length *L*, hinged at *A*. Force at hinge is $\bar{F}$, making an angle θ with the beam. *T* is the tension in the rope, making an angle 45° with the beam.

Taking moments of all forces, about *A*,



$500 \times 0.5L + W \times 0.6L = T \sin 45° \cdot L$

$W = (T \sin 45° - 250) \times \frac{1}{0.6}$

$W \to W_{\max'}$ when $T \to T_{\max} = 1800N$

$\therefore \quad W_{\max} = (1800 \times 0.707 - 250) \times \frac{10}{6}$

$= (70.7 \times 18 - 250) \times \frac{10}{6} = (1272.6 - 250) \times \frac{10}{6} = 1704N \quad \Rightarrow \quad 170.4 \text{ kg}$

If *P* is moved to *O*, $(500 + W)0.5L = 1800 \times \frac{L}{\sqrt{2}}$

$\therefore \quad W = \frac{3600}{\sqrt{2}} - 500 = 2045.2N \Rightarrow 204.5 \text{ kg}$ [Increased from 170.4 kg to 204.5 kg].

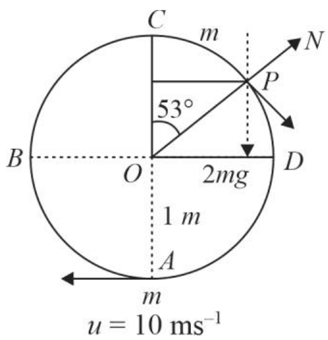
For 60 kg load at mid point

$g(50 + 60)0.5L = T \sin\theta \cdot L$

$T = 110 \times 10 \times 0.5 \times \sqrt{2} = 550 \times 1.414$

$T = 778N$

10.(AD) Taking potential energy at $A = 0$, the speed of the body projected at *A* when it reaches *C* is u_1,



$\frac{1}{2}mu^2 = \frac{1}{2}mu_1^2 + mg \times 2$

$u_1^2 = u^2 - 4g = 100 - 40 = 60$

$u_1 = \sqrt{60} = 2\sqrt{15} \text{ ms}^{-1}$

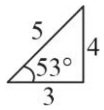
Let u_2 be the speed of combined mass 1 kg after collision.

$0.5u_1 = 1 \times u_2$

$u_2 = \sqrt{15} \text{ ms}^{-1} = 3.87 \text{ ms}^{-1}$

At P, $\theta = 53°$, let u_3 be the speed at P, then

$$2mg\cos\theta - N = \frac{2m \cdot u_3^2}{R}$$



If it leaves the track, $N = 0$

$$u_3^2 = gR\cos\theta = 10 \times 1 \times \frac{3}{5} = 6$$

At C, Kinetic Energy of $(2m) = \frac{1}{2} \times 2m \times u_2^2 = \frac{15}{2} J$

Potential Energy of $(2m) = 2m \times g \times 2 = 20J$

$\therefore$ total energy at $C = E_C =$ Kinetic Energy + Potential Energy $= \frac{55}{2} = 27.5J$

At P, Kinetic Energy of $(2m) = \frac{1}{2} \times 1 \times u_3^2 = \frac{6}{2} = 3J$

Potential Energy of $(2m) = 2m \times g \times (R + R\cos\theta)$ $= 10\left(1 + \frac{3}{5}\right) = 16J$

$\therefore$ total energy at $P = E_P = 3 + 16 = 19J$

Net work done on the body = Change in kinetic energy $= K_2 - K_1 = 3 - 7.5 = -4.5J$

$$(W.D.)_{\text{Conservative}} + (W.D.)_{\text{Nonconservative}} + \text{other} = (K_2 - K_1)$$

$$(W.D.)_{NC} = K_2 - K_1 + V_2 - V_1 \quad \text{[No W.D. by other forces]}$$

$$= E_2 - E_1$$

Here, W.D. by friction $= E_P - E_C = 19 - 27.5 = -8.5J$

$\therefore$ W.D. by the body against friction $= 8.5J$

11.(BC) $a = \dfrac{g\sin\theta}{1 + \dfrac{k^2}{R^2}}$, $t = \sqrt{\dfrac{2s}{a}} = \sqrt{\dfrac{2h}{a\sin\theta}}$ $(\because s\sin\theta = h)$

For A, $\frac{2}{3}mR^2 = mk^2 \Rightarrow \frac{k^2}{R^2} = \frac{2}{3}$ $\therefore$ $a_A = \dfrac{g\sin 60°}{1 + \frac{2}{3}} = \dfrac{3\sqrt{3}g}{10}$; $t_A = \sqrt{\dfrac{2h}{\frac{3\sqrt{3}g}{10} \times \frac{\sqrt{3}}{2}}} = \sqrt{\dfrac{40h}{9g}}$

For B, $\frac{2}{5}mR^2 = mk^2 \Rightarrow \frac{k^2}{R^2} = \frac{2}{5}$ $\therefore$ $a_B = \dfrac{g\sin 30°}{1 + \frac{2}{5}} = \dfrac{5g}{14}$

$$t_B = \sqrt{\frac{2h}{\frac{5g}{14} \times \frac{1}{2}}} = \sqrt{\frac{56h}{5g}} \qquad \therefore \quad t_B > t_A$$

$$k_A = k_B = mgh; \qquad k_A = \frac{1}{2}mv_A^2 + \frac{1}{2}\frac{2}{3}mR^2\left(\frac{v_A}{R}\right)^2 = \frac{5}{6}mv_A^2$$

$$k_B = \frac{1}{2}mv_B^2 + \frac{1}{2}\frac{2}{5}mR^2\left(\frac{v_B}{R}\right)^2 = \frac{7}{10}mv_B^2 \qquad \therefore \quad \frac{5}{6}mv_A^2 = \frac{7}{10}mv_B^2 \Rightarrow v_B = \sqrt{\frac{25}{21}}v_A$$

12.(AC) $a_x = \frac{F}{m}\cos(\omega t)$; $a_y = \frac{F}{m}\sin(\omega t)$

$v_x = \frac{F}{m\omega}\sin(\omega t)$ **;** $v_y = \frac{F}{m\omega}(1-\cos(\omega t))$

Speed $v = \sqrt{v_x^2 + v_y^2}$ $= \frac{2F}{m\omega}\sin\left(\frac{\omega t}{2}\right)$

$v = 0 \quad \Rightarrow \quad \frac{\omega t}{2} = n\pi$ or $t = \frac{2n\pi}{\omega}$

Distance in $(0-t) = \int_0^{\frac{2\pi}{\omega}} v\,dt$ $= \frac{8F}{m\omega^2}$

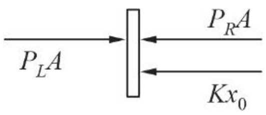
Average speed $= \frac{8F \times \omega}{m\omega^2 \times 2\pi} = \frac{4F}{\pi m\omega}$

$x = \frac{F}{m\omega^2}\left(1-\cos(\omega t)\right)$; $y = \frac{Ft}{m\omega} - \frac{F}{m\omega^2}\sin(\omega t)$

13.(AC) $|W_L| = |W_R| + \frac{1}{2}kx_0^2$

$\Delta Q =$ change in internal energies of gases on the two sides $+\frac{1}{2}kx_0^2$

$P_L = P_R + \frac{kx_0}{A}$



14.(ABCD)

Finally velocity of the cart and the block will be same

$mv = (m + 32m)v_f \quad \Rightarrow \quad v_f = \frac{v}{33}$

Velocity of the block w.r.t cart after five collision $= \frac{v}{32}$ (left)

So, $m\left(-\frac{v}{32} + v_C\right) + 32m(v_C) = mv$; $33mv_C = \frac{33mv}{32}$

$v_C = \frac{v}{32}$ (twowards right)

So after five collisions velocity of block w.r.t ground $= -\frac{v}{32} + \frac{v}{32} = 0$ time in five collisions

$= \frac{d}{v} + \frac{2d}{v} + \frac{4d}{v} + \frac{8d}{v} + \frac{16d}{v} = \frac{31d}{v}$

15.(B) **(I)** $\frac{dv}{dt} = \text{constant} \quad a \propto x$

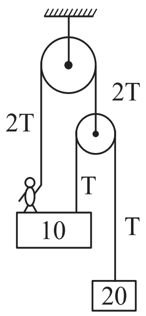
(II) $\frac{dv^2}{dx} = \frac{2vdv}{dx} = \text{constant} \Rightarrow a = \text{constant}$

(III) $a = \text{constant}$

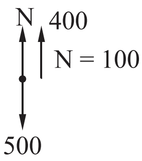
(IV) $\frac{dv}{dt^2} = \text{constant} \Rightarrow \quad \frac{dv}{dt} \propto t$

16.(A) In equilibrium

$\Rightarrow \quad T = 200 \Rightarrow 3T = W + 100 \Rightarrow \quad W = 500$

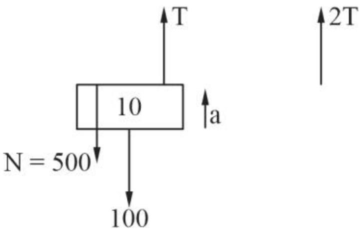


For man



If $N = 500$

Platform **Man**



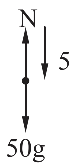
$T - 600 = 10a; \quad T = 50a$

$\Rightarrow \quad 15a = 600 \;;\; a = 40 \text{ m/s}^2$

$2T = 50 \times 40 = 2000N$

$\Rightarrow$ If string is let go:

$a = \frac{(60-20)10}{80} = 5 \text{ m/s}^2$



$500 - N = 5 \times 50$

$N = 250$ Newton

17.(A) $V_3 = \sqrt{2 \times g \times 8} = 4\sqrt{10}\, m/s$

$V_2 = \frac{V_3}{3} = \frac{4}{3}\sqrt{10}$

Applying Bernoulli's between pts 2 & 3

$$P_2 + \frac{1}{2}\rho \times \left(\frac{4}{3} \times \sqrt{10}\right)^2 = P_0 + \frac{1}{2}\rho \times \left(4\sqrt{10}\right)^2$$

$$P_2 - P_0 = \frac{1}{2}\rho \times \left(4\sqrt{10}\right)^2 \times \frac{8}{9} = 1000 \times 160 \times \frac{4}{9} = 71111 \text{ Pa}$$

$$P_{bottom} - P_0 = \rho \times g \times 5 = 50000 Pa$$

18.(B) (I) Process is isobaric

So $C_v = \frac{5}{2}R,\ C_P = \frac{7}{2}R$

$\Delta Q = \frac{7}{2}RT_0,\ \Delta U = \frac{5}{2}RT_0$

$\Delta W = RT_0$

(II) $\frac{P}{T} = \text{constant, isochoric process}$

$C = C_v = \frac{3}{2}R$

$\Delta Q = \Delta U = \frac{3}{2}RT_0$

$\Delta W = 0$

(III) $C_v = \frac{R}{1.5-1} = 2R$

$VT = \text{constant};\ PV^2 = \text{constant};\ \frac{P}{\rho_2} = \text{constant}$

$V + T\frac{dV}{dT} = 0;\qquad \frac{dV}{dT} = -\frac{V}{T}$

$\frac{P}{n}\frac{dV}{dT} = -\frac{PV}{nT} = -R;\quad C = C_v - R = R$

$\Delta Q = RT_0,\ \Delta U = 2RT_0;\ \Delta W = -RT_0$

(IV) $\gamma = 1 + \frac{2}{4} = \frac{3}{2};\ C_v = \frac{R}{\frac{3}{2}-1} = 2R$

$PT = \text{constant};\ P^2V = \text{constant};\ \frac{P^2}{\rho} = \text{constant}$

$\Delta U = 2RT_0;\quad \Delta W = 2RT_0;\quad \Delta Q = 4RT_0$

CHEMISTRY

1.(448) Given, $V = 4L,\ T = 273K,\ P = 2.8\,atm,\ n = 0.4\,mole$

$$PV = nRT$$

$$(2.8)4 = n(0.0821)273$$

$$\Rightarrow \quad n_{Total} = 0.5\,mole \quad \Rightarrow \quad n_{unknown} = 0.1\,mole$$

$$r_{effusion} \propto \frac{n'}{t}\ ; \quad \frac{r_{N_2}}{r_{unknown}} = \frac{0.4/10}{0.1/10} = \sqrt{\frac{M^o_{unknown}}{M_{N_2}}}$$

$$(4/1)^2 = \frac{M^o_{unknown}}{28} \quad \Rightarrow \quad M^o_{unknown} = 28 \times 16 = 448\,g\,mol^{-1}$$

2.(74.80) $C(s) + 2H_2(g) \longrightarrow CH_4(g)$

$$\Delta H = -393.5 - 285.8 \times 2 + 890.3 = -74.8\ kJ/mol$$

So, heat of formation (magnitude) = 74.8 kJ/mole.

3.(80) Eq. of Fe = Eq. of H_2

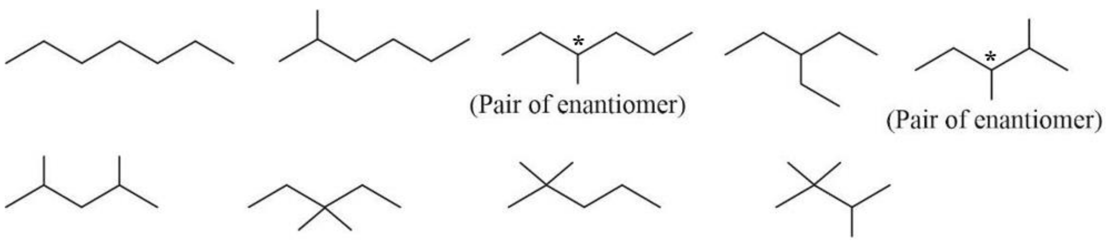
$$\frac{W}{56} \times 3 = \frac{6}{2} \times 2, \qquad W = \frac{6 \times 56}{3}, \qquad \%\ of\ Fe = \frac{6 \times 56}{3} \times \frac{100}{140} = 80$$

4.(11.40) Eq. of HNO_3 used = Eq. of NaOH used.

Maximum 0.2 equivalent of both can be neutralized

Heat evolved = $57 \times 0.2 = 11.4\,kJ$

5.(11)



6.(8.30) Sodium bicarbonate is a salt of amphoteric anion. Its pH is given by expression

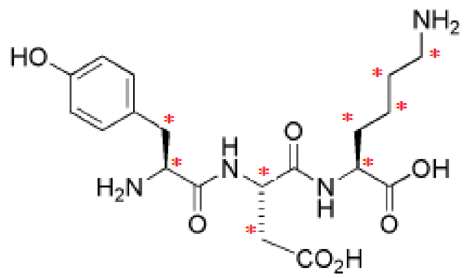
$$pH = \frac{pKa_1 + pKa_2}{2} = \frac{6.3 + 10.3}{2} = 8.3$$

7.(37) $Z_1 = 10$, $Z_2 = 9$, $Z_3 = 18$

(C of C = O and benzene ring are sp^2

Two lone pair on every O and one lone pair on every N)

* sp^3 carbon atom (in the given figure)



8.(6) Electronic configuration of As, $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^{10}, 4p^3$. Electrons of 3p have n = 3 and l = 1.

9.(ABC)

Given, 0.01 M each of Zn^{2+}, Mg^{2+}, Mn^{2+} and 0.1 M H_2S

For ppt. of MnS : $Q_{sp} > K_{sp}$

$$[S^{2-}] > 10^{-20} \rightarrow 10^{-20} < [S^{2-}] \le 10^{-16}$$

For ppt. of ZnS :

$$[S^{2-}] > 10^{-16} \rightarrow 10^{-16} < [S^{2-}] \le 10^{-10}$$

For ppt. of MgS :

$$[S^{2-}] > 10^{-10}$$

$$H_2S \rightleftharpoons 2H^+ + [S^{2-}]$$

$$K_a = \frac{[H^+]^2[S^{2-}]}{[H_2S]} \quad \Rightarrow \quad [S^{2-}] = \frac{K_a[H_2S]}{[H^+]^2} = \frac{10^{-22}}{[H^+]^2}$$

(A) When $pH = 1$ to 3

$[H^+] = 10^{-3}$ to 10^{-1}

So, $[S^{2-}] = 10^{-20}$ to 10^{-16} $\Rightarrow$ Hence, MnS will get precipitated.

(B) When $pH > 3$

$[H^+] < 10^{-3}$

So, $[S^{2-}] > 10^{-16}$ $\Rightarrow$ Hence, ZnS will get precipitated.

(C) When $pH > 6$

$[H^+] < 10^{-6}$

So, $[S^{2-}] > 10^{-10}$

$\Rightarrow$ Hence, MgS will get precipitated.

10.(BCD)

Number of e^- in a subshell

S = 3, P = 9, d = 15, f = 21

Noble gas configuration ns^3np^9 i.e. 12 valance electrons.

Electronic configuration of X_2 molecule.

$\sigma 1s^3, \sigma^* 1s^3, \sigma 2s^3$

$\sigma^* 2s^3, \pi 2p_x^3 = \pi 2p_y^3$

$\sigma 2p_z^3, \pi^* 2p_X^2 = \pi^* 2p_y^1$

Bond order of X_2 is 2 (1 σ bond and 1 π bond)

11.(ABCD)

$$\underset{(10\,atm,\ 300\,K)}{A} \xrightarrow{\text{reversible}} \underset{(5\,atm,\ 300\,K)}{B} \xrightarrow{P_{ext}=1\,atm} \underset{(1\,atm,\ 300\,K)}{C}$$

(A) $W_T = W_{AB} + W_{BC}$

$$= -nRT\,\ell n\left(\frac{P_1}{P_2}\right) - P_{ext}\left(\frac{nRT}{P_2} - \frac{nRT}{P_1}\right)$$

$$= -1\times 2\times 300\times \ell n 2 - 1\times 1\times 2\times 300\left(\frac{1}{1}-\frac{1}{5}\right)$$

$W_T = -900\,cal$

(B) $(\Delta S)_{AB} = nR\,\ell n\left(\frac{P_1}{P_2}\right) = 1\times 2\times \ell n 2$

$(\Delta S)_{AB} = 1.4\ cal/K$

(C) $dG = Vdp - SdT \qquad (SdT = 0)$

$dG = Vdp$

$\Delta G = nRT\,\ell n\left(\frac{P_2}{P_1}\right) = 1\times 2\times 300\times \ell n\left(\frac{1}{2}\right)$

$\Delta G = -420\,cal$

(D) $\Delta S_{surr} = -\frac{q_{system}}{T}$

$\Delta U = q + w \quad \Rightarrow \quad q = -w \qquad (\Delta U = 0)$

$\Delta S_{surr} = \frac{w_{irr}}{T}$; $\Delta S_{surr} = \frac{-P_{ext}\,\Delta V}{T}$

$\Delta S_{surr} = -P_{ext}nR\left(\frac{1}{P_2}-\frac{1}{P_1}\right) = -1\times 1\times 2\left(\frac{1}{1}-\frac{1}{5}\right)$

$(\Delta S_{surr})_{BC} = -1.6\ cal/K$

12.(ABC) Relations given in options A, B and C are true using the equation of states under isothermal and adiabatic conditions and the respective P – V diagrams.

13.(AB) O_2 : $\sigma 1s^2\,\sigma^*1s^2\,\sigma 2s^2\,\sigma^*2s^2\,\sigma 2p_z^2\,\pi 2p_x^2 = \pi 2p_y^2\,\pi^*2p_x^1 = \pi^*2p_y^1$

So HOMO (highest occupied molecular orbital) is π^*2p_x or π^*2p_y

N_2 : $\sigma 1s^2\,\sigma^*1s^2\,\sigma 2s^2\,\sigma^*2s^2\,\pi 2p_x^2 = \pi 2p_y^2\,\sigma 2p_z^2\ \pi^*2p_x^0 = \pi^*2p_y^0$

So LUMO (lowest unoccupied molecular orbital) is π^*2p_x or π^*2p_y

C_2 : $\sigma 1s^2\,\sigma^*1s^2\,\sigma 2s^2\,\sigma^*2s^2\,\pi 2p_x^2 = \pi 2p_y^2$

C_2 is stable as bond order is positive

C_2^{2-} : $\sigma 1s^2\,\sigma^*1s^2\,\sigma 2s^2\,\sigma^*2s^2\,\pi 2p_x^2 = \pi 2p_y^2\,\sigma 2p_z^2$

C_2^{2-} is stable due to positive value of bond order.

14.(BCD) $HCl + NaOH \longrightarrow NaCl + H_2O,$ so mixture can't acts as buffer because both are strong electrolytes.

$CH_3COONa + HCl \longrightarrow CH_3COOH + NaCl,$ so mixture can be a buffer

$NH_4OH + HCl \longrightarrow NH_4Cl + H_2O,$ so mixture can be a buffer

$NH_4Cl + NaOH \longrightarrow NH_4OH + NaCl,$ so mixture can be a buffer

15.(A) **I-P; II-S ; III – R ; IV - Q**

(I)	Rev. Cooling of ideal gas at constant volume:	w = 0 q = –ve $\Delta U = -$ ve
(II)	Rev. isothermal expansion of ideal gas	w = –ve q = +ve $\Delta U = 0$
(III)	Adiabatic is free expansion of ideal gas into vacuum	w = 0 q = 0 $\Delta U = 0$
(IV)	Rev. isothermal compression of an ideal gas.	w = +ve q = –ve $\Delta U = 0$

16.(C) a : $He < CH_4 < SO_2$

vander Waal's constant

$V_{rms} \propto \frac{1}{\sqrt{M_0}}$ $\quad\therefore\quad$ $V_{rms} = He > CH_4 > SO_2$

$(K.E.)_{Per\,mole} = \frac{3}{2}RT$ $\quad\therefore\quad$ $(K.E.) \Rightarrow He = CH_4 = SO_2$

Rate of diffusion $\propto \frac{P}{\sqrt{M_0}}$

17.(A) In compound (I), Resonance effect is present and aromaticity is also present (cyclic + completely conjugated + planar and 6π delocalized electrons.

In compound (II), + I effect is present because of methyl group, Hyperconjugation is present because of α-hydrogen and resonance effect is also present.

In compound (III), Resonance effect is present and the compound is aromatic as it is cyclic, completely conjugated and planar and have 6π delocalized electrons. – I effect is also present because of Chlorine.

In compound (IV), Resonance effect is present and the compound is aromatic because it is cyclic, completely conjugated and planar and a close loop of 6π delocalized electrons is present. But the resonating structure with charged ring will be not aromatic.

18.(C) **(P)** Wurtz reaction

(Q) Allylic substitution followed by dehydrohalogenation

(R) Dehydration of alcohol in the presence of conc. H_2SO_4 / Δ

(S) $NH_2–NH_2$ $(OH)^-$ reduces ketone to alkane [Wolff-Kishner reaction]

MATHEMATICS

1.(12) $f(x) = x$ Now, Put $x = \cos\theta, \Rightarrow \sqrt{x^2 - 1} = i\sin\theta$

$$\left(x+\sqrt{x^2-1}\right)^{10} + \left(x-\sqrt{x^2-1}\right)^{10} = (\cos\theta + i\sin\theta)^{10} + (\cos\theta - i\sin\theta)^{10}$$

$$= 2\cos(10\theta) = 2f_{10}(\cos\theta) = 2f_{10}(x)$$

2.(0.56) $\left(n^2-1\right)^3 = (n+1)^3(n-1)^3$

$$(n+1)^3 - (n-1)^3 = 6n^2 + 2$$

$$\frac{3n^2+1}{\left(n^2-1\right)^3} = \frac{1}{2}\frac{6n^2+2}{\left(n^2-1\right)^3} = \frac{1}{2}\left(\frac{(n+1)^3-(n-1)^3}{(n+1)^3(n-1)^3}\right)$$

$$= \frac{1}{2}\left(\frac{1}{(n-1)^3} - \frac{1}{(n+1)^3}\right)$$

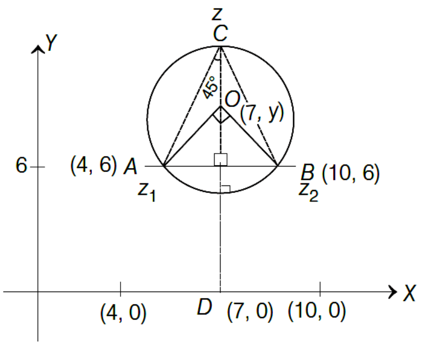
$$S = \frac{1}{2}\left[\left(\frac{1}{1^3}-\frac{1}{3^3}\right)+\left(\frac{1}{2^3}-\frac{1}{4^3}\right)+\left(\frac{1}{3^3}-\frac{1}{5^3}\right)+\left(\frac{1}{4^3}-\frac{1}{6^3}\right)+\ldots\right]$$

$$= \frac{1}{2}\left(1+\frac{1}{8}\right) = \frac{9}{16} \Rightarrow 16S = 9$$

3.(4.24) Since, $z_1 = 10 + 6i, z_2 = 4 + 6i$

and $\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{4}$ represents locus of z is a circle shown as from the figure whose centre is (7, y) and $\angle AOB = 90°, \Rightarrow OD = 6 + 3 = 9$

$\therefore$ Centre = (7, 9) and radius $= \frac{6}{\sqrt{2}} = 3\sqrt{2}$



$\Rightarrow$ Equation of circle is $|z - (7 + 9i)| = OC = 3\sqrt{2}$

4.(1210) Here, $a+b=10c$ and $c+d=10a$

$\Rightarrow \quad (a-c)+(b-d)=10(c-a)$

$\Rightarrow \quad (b-d)=11(c-a) \quad \ldots(i)$

Since, 'c' is the roots of $x^2-10ax-11b=0$

$\Rightarrow \quad c^2-10ac-11b=0 \quad \ldots(ii)$

Similarly, 'a' is the root of

$x^2-10cx-11d=0$

$\Rightarrow \quad a^2-10ca-11d=0 \quad \ldots(iii)$

On subtracting Eq. (iv) from Eq. (ii), we get

$\left(c^2-a^2\right)=11(b-d) \quad \ldots(iv)$

$\therefore \quad (c+a)(c-a)=11\times 11(c-a) \quad$ [from Eq. (i)]

$\Rightarrow \quad c+a=121$

$\therefore \quad a+b+c+d=10c+10a$

$=10(c+a)=1210$

5.(12) $\alpha=\frac{1+\sqrt{5}}{2}, \beta=\frac{1-\sqrt{5}}{2}$

$a_4=a_3+a_2$

$=2a_2+a_1=3a_1+2a_0$

$28=p(3\alpha+2)+q(3\beta+2)$

$28=(p+q)\left(\frac{3}{2}+2\right)+(p-q)\left(\frac{3\sqrt{5}}{2}\right)$

$\therefore \quad p-q=0$ and $(p+q)\times\frac{7}{2}=28$

$\Rightarrow \quad p+q=8 \quad \Rightarrow \quad p=q=4$

$\therefore \quad p+2q=12$

6.(0) $\sin^3 x\sin 3x=\sin^3 x(3\sin x-4\sin^3 x)=3\sin^4 x-4\sin^6 x=3(1-\cos^2 x)^2-4(1-\cos^2 x)^3$

$=3\left(1+t^4-2t^2\right)-4\left(1-t^6-3t^2+3t^4\right)$ (where $t=\cos x$), Now $\quad C_0+C_2+C_4+C_6=0$

7.(9) $\sin 5\theta\cos 3\theta=\sin 9\theta \cdot \cos 7\theta$

$\frac{\sin 8\theta+\sin 2\theta}{2}=\frac{\sin 16\theta+\sin 2\theta}{2} \Rightarrow \sin 8\theta=\sin 16\theta$, $\sin 8\theta=2\sin 8\theta\cos 8\theta$

$\Rightarrow \quad \sin 8\theta=0 \Rightarrow \theta=0, \frac{\pi}{8}, \frac{2\pi}{8}, \frac{3\pi}{8}, \frac{4\pi}{8}$ or $\cos 8\theta=\frac{1}{2}$

$\Rightarrow \quad \theta=\frac{\pi}{24}, \frac{5\pi}{24}, \frac{7\pi}{24}, \frac{11\pi}{24} \quad \therefore$ 9 solutions

8.(0.22) Let $S = \frac{2}{3} - \frac{5}{6} + \frac{2}{3} - \frac{11}{24} + ...\infty$

Which can be expressed as: $S = \frac{2}{3} + \frac{5}{3}\left(\frac{-1}{2}\right) + \frac{8}{3}\left(\frac{-1}{2}\right)^2 + \frac{11}{3}\left(\frac{-1}{2}\right)^3 + ...\infty$ .....(i)

It is a AGP series with $r = \frac{-1}{2}$. Multiplying both sides by $-\frac{1}{2}$, we get:

$$-\frac{1}{2}S = \frac{2}{3}\left(\frac{-1}{2}\right) + \frac{5}{3}\left(\frac{-1}{2}\right)^2 + \frac{8}{3}\left(\frac{-1}{2}\right)^3 + ...\infty \quad .....(ii)$$

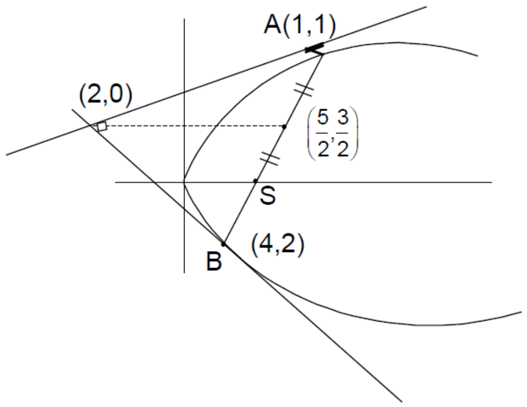
Subtracting (ii) from (i), we have $\frac{3}{2}S = \frac{2}{3} + \frac{3}{3}\left(\frac{-1}{2}\right) + \frac{3}{3}\left(\frac{-1}{2}\right)^2 + \frac{3}{3}\left(\frac{-1}{2}\right)^3 + ...\infty$

$$\frac{3}{2}S = \frac{2}{3} - \left[\frac{1}{2} - \frac{1}{2^2} + ...\infty\right] \Rightarrow \frac{3}{2}S = \frac{2}{3} - \frac{\frac{1}{2}}{1 - \left(-\frac{1}{2}\right)} \Rightarrow \frac{3}{2}S = \frac{1}{3} \Rightarrow S = \frac{2}{9}$$

9.(AC) $\because$ Tangents are $\perp r$. so, they intersect on directrix.

Point of intersection = (2, 0) mid-point of (1, 1) & (4, 2) is $\left(\frac{5}{2}, \frac{3}{2}\right)$

Slope of axis $= \frac{\frac{3}{2} - 0}{\frac{5}{2} - 2} = 3$



Equation of directrix, $y = -\frac{1}{3}(x - 2) \Rightarrow x + 3y = 2$

AB is focal chord,

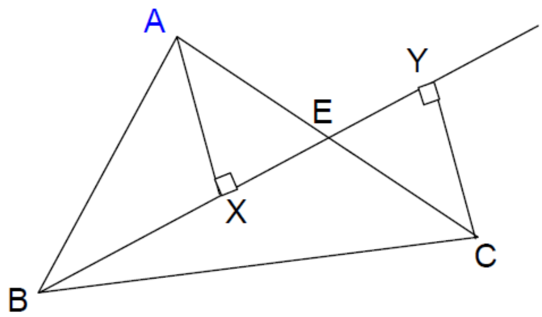
$BS = (\perp_r \text{ distance from } B \text{ on directrix}) = \frac{4 + 6 - 2}{\sqrt{10}} = \frac{8}{\sqrt{10}}$

$AS = (\perp_r \text{ distance from } A \text{ on directrix}) = \frac{1 + 3 - 2}{\sqrt{10}} = \frac{2}{\sqrt{10}}$

So, focus divides *AB* in 1 : 4 ratios. So $S = \left(\frac{8}{5}, \frac{6}{5}\right)$

10.(ABD) $\Delta AXE \simeq \Delta CYE$

so, $ar(\Delta AXE) = ar(\Delta CYE) = \Delta_1$



$ar(\Delta BYC) = ar(\Delta BEC) + \Delta_1$

$4\Delta_1 = ar(\Delta BEC) + \Delta_1$

$ar(\Delta BEC) = 3\Delta_1 = ar(\Delta ABE) = ar(\Delta AXB) + ar(\Delta AXE)$

$\Rightarrow ar(\Delta AXB) = 2\Delta_1$

$ar(\Delta ABC) = 2ar(\Delta BEC) = 6\Delta_1$

11.(AC) $\frac{A}{a}, \frac{B}{b}, \frac{C}{c} HP$

$\Rightarrow \frac{2b}{B} = \frac{a}{A} + \frac{c}{C}$

$\Rightarrow 2bB = aC + cA$

$\Rightarrow aB + cB = aC + cA$

$\Rightarrow a[B - C] = c[A - B]$

So, $r = \frac{c}{a}$

(B) for $r = d$

$$\left(\frac{A}{a} + \frac{C}{c}\right)\frac{b}{B} = \frac{b}{ar} + \frac{rb}{c} = \frac{bc + r^2ab}{acr}$$

$$= \frac{(a+r)(a+2r) + r^2a(a+r)}{a(a+2r)r}$$

$$= \frac{a^2 + 3ar + 2r^2 + a^2r^2 + ar^3}{a(a+2r)r} = 2$$

$\Rightarrow a^2 + 3ar + 2r^2 + a^2r^2 + ar^3 = 2a^2r + 4ar^2$ will not hold for all a and r.

$\Rightarrow \frac{a}{A}, \frac{b}{B}, \frac{c}{C}$ cannot be in H.P if $r = d$

(C) $\frac{A^2}{a}, \frac{B^2}{b}, \frac{c^2}{c}$ are in HP $\Rightarrow \frac{2b}{B^2} = \frac{a}{A^2} + \frac{c}{C^2}$

$\Rightarrow 2bB^2 = aC^2 + cA^2$

$\Rightarrow aB^2 + cB^2 = aC^2 + cA^2$

$\Rightarrow a\left(B^2 - C^2\right) = c\left(A^2 - B^2\right)$

$\Rightarrow r = \sqrt{\frac{c}{a}}$

(D) $2 = \left(\frac{A^2}{a} + \frac{C^2}{c}\right)\frac{b}{B^2} = \frac{b}{ar^2} + \frac{r^2 b}{c}$

$= \frac{a+d}{ad} + \frac{d\left(a+d\right)}{a+2d} = \frac{a^2 + 3ad + 2d^2 + a^2d^2 + ad^3}{ad\left(a+2d\right)}$

$\Rightarrow 2a^2d + 4ad^2 = a^2 + 3ad + 2d^2 + a^2d^2 + ad^3$ will not always hold for all *a* and *d*.

$\therefore$ D is incorrect.

12.(ABCD) $|f| = 3 \Rightarrow f = \pm 3$

$f^2 - c - 0 \Rightarrow c = 9$

$2\sqrt{g^2 - c} = 8$

$\Rightarrow g^2 - c = 4^2 \Rightarrow g^2 - c = 16$

$\Rightarrow g^2 = 25$

$\Rightarrow g = \pm 5$

13.(BD) $f\left(0\right) = r$ is odd. Let $r = 2n+1, n \in I$

$f\left(-1\right) = -1 + p - q + 2n + 1 = p - q + 2n$ is odd

$\Rightarrow$ exactly one of *p*, *q* is odd

$f(1) = 1 + p + q + 2n + 1 = p + q + 2n + 2$ is odd

If possible suppose $\alpha, \beta, \gamma, \in I$ be zeros of $f(x)$.

$\Rightarrow x^3 + px^2 + qx + r = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$\Rightarrow r = -\alpha\beta\gamma \Rightarrow \alpha, \beta, \gamma$ are odd integers

$\Rightarrow p = -(\alpha + \beta + \gamma)$ is odd

and $q = \alpha\beta + \beta\gamma + \gamma\alpha$ is also odd.

It is a contradiction. Hence $f(x) = 0$

Cannot have three integer roots.

14.(ABC) The tangent $3x+4y-25=0$ is tangent at vertex and axis is $4x-3y=0$

So, $PS=a=5$ Latus rectum $=AB=20$

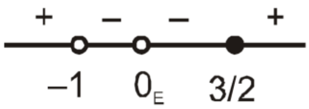
15. (A) (P) $\log_{\sin x}\left(\log_3\left(\log_{0.2}x\right)\right)<0 \quad =\log_{\sin x}1$

$$\Rightarrow \quad \log_3\left(\log_{0.2}x\right)>1 \;\Rightarrow\; \log_{0.2}x>3=\log_{0.2}(0.2)^3$$

$$\Rightarrow \quad 0<x<(0.2)^3 \;\Rightarrow\; 0<x<\frac{1}{125}$$

(Q) $$\frac{\left(e^x-1\right)(2x-3)\left(x^2+x+2\right)}{(\sin x-2)x(x+1)}\le 0$$

$$\Rightarrow \quad \frac{\left(e^x-1\right)(x-3/2)}{x(x+1)}\ge 0$$



$$\Rightarrow \quad x<-1 \text{ or } x\ge\frac{3}{2} \;\Rightarrow\; x\in(-\infty,-1)\cup\left[\frac{3}{2},\infty\right)$$

(R) $\left|2-|x-1|\right|\le 2 \;\Rightarrow\; \left||x-1|-2\right|\le 2 \;\Rightarrow 0\le|x-1|\le 4 \;\Rightarrow\; -3\le x\le 5$

(S) $\left|\sin\left(3x-4x^3\right)\right|\le 0 \;\Rightarrow\; \sin\left(3x-4x^3\right)=0 \;\Rightarrow\; x=0$

16.(B) We have $Z^n-1=(z-1)\left(Z-e^{\frac{i2\pi}{n}}\right)\text{.....}\left(Z-e^{\frac{i2\pi}{n}(n-1)}\right)$

$$1+z+\ldots+z^{n-1}=\left(Z-e^{\frac{i2\pi}{n}}\right)\text{....}\left(Z-e^{\frac{i2\pi}{n}(n-1)}\right) \qquad \text{.... (1)}$$

Put z = 1 and take modulus $n=2^{n-1}\sin\frac{\pi}{n}\sin\frac{2\pi}{n}\text{......}\sin(n-1)\;\frac{\pi}{n}$

(P) If n = 21

$$21=2^{20}\sin\frac{\pi}{21}\sin\frac{2\pi}{21}\text{......}\sin\frac{9\pi}{21}.\sin\frac{10\pi}{21}\text{.....}\sin\frac{20\pi}{21}$$

$$21=\left(\sin\frac{\pi}{21}\text{.....}\sin\frac{10\pi}{21}\right)^2.2^{20} \qquad \therefore \sin\frac{\pi}{21}\text{......}\sin\frac{10\pi}{21}=\frac{\sqrt{21}}{2^{10}}$$

(Q) If n = 22

$$22=2^{21}\left(\sin\frac{\pi}{22}\sin\frac{2\pi}{22}\text{........}\sin\frac{10\pi}{22}\right)^2$$

$$\therefore \left(\sin\frac{\pi}{22}\text{.....}\sin\frac{10\pi}{22}\right)^2=\frac{22}{2^{21}}=\frac{11}{2^{20}} \qquad \therefore \sin\frac{\pi}{22}\sin\frac{2\pi}{22}\text{.....}\sin\frac{10\pi}{22}=\frac{\sqrt{11}}{2^{10}}$$

(R) Again put $z = -1$ is...........(1) and take modulus

$$1 = \left|\cos\frac{\pi}{n}\cos\frac{2\pi}{n}.....\cos(n-1)\frac{\pi}{n}\right|2^{n-1}$$

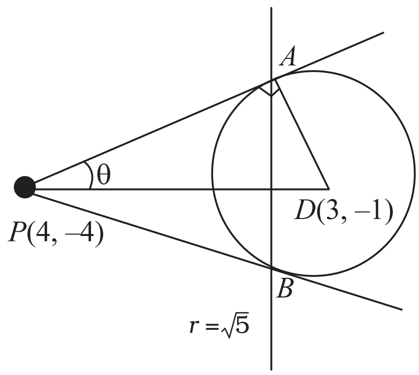
$$n = 21$$

$$1 = \left|\cos\frac{\pi}{21}\cos\frac{2\pi}{21}.....\cos\frac{20\pi}{21}\right|2^{20} \qquad \therefore \cos\frac{\pi}{21}......\cos\frac{10\pi}{21} = \frac{1}{2^{10}}$$

(S) $$\left(\sin\frac{\pi}{22}.\sin\frac{2\pi}{22}.....\sin\frac{10\pi}{22}\right) = \frac{\sqrt{11}}{2^{10}}$$

$$\left(\cos\frac{10\pi}{22}\cos\frac{9\pi}{22}.....\cos\frac{\pi}{22}\right) = \frac{\sqrt{11}}{2^{10}} \qquad \because \sin\theta = \cos(90-\theta)$$

17.(C)



Chord of contact of P is $S_1 = 0,\ x - 3y - 11 = 0$

Length of $AB = 2\sqrt{r^2 - d^2} = \sqrt{10}$

$$\sin\theta = \frac{AC}{PC} = \frac{\sqrt{5}}{\sqrt{10}} = \frac{1}{\sqrt{2}} \Rightarrow \tan\theta = 1$$

Area of $\Delta\ PAB = \frac{1}{2}\times\sqrt{10}\times\frac{5}{\sqrt{10}} = \frac{5}{2}$

Let slope be m

Tangent line be $mx - y = 4m + 4$, $r = d$

$$\sqrt{5} = \frac{|3m+1-4m-4|}{\sqrt{1+m^2}} \Rightarrow 2m^2 - 3m - 2 = 0\ m = \frac{-1}{2},\ 2 \Rightarrow |m_1 - m_2| = \frac{5}{2}$$

18.(D) $P \to \theta = \frac{\pi}{6}$

$Q \to$ For any k, it is rectangular hyperbola.

$R \to a = 0, 1, 2, 3, 4$

$S \to$ compare with $xy = c^2$

Tangent at (x_1, y_1) is $\frac{x}{x_1} + \frac{y}{y_1} = 2 \Rightarrow K = 2c^2 = 2(1 + \sin^2\theta)$